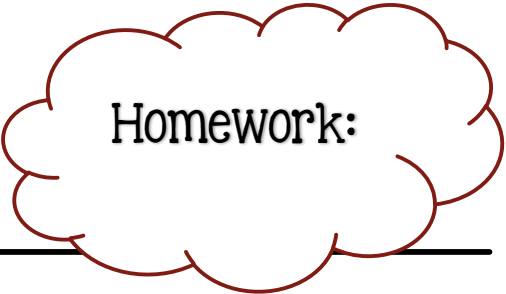




# Calculus 1.1 Understanding Limits Numerically and Graphically

Limits are one of the BIG Ideas in the AP Calculus Curriculum

Working Definition: The value of  $f(x)$  as  $x$  approaches a certain number when approaching from either direction. The limit doesn't have to equal  $f(x)$  at  $x$ .



## Example 1

Graph and Analyze

$$f(x) = \frac{3x^2 - 8x - 3}{x^2 - 9}$$

Factor and Simplify

Hole(s)

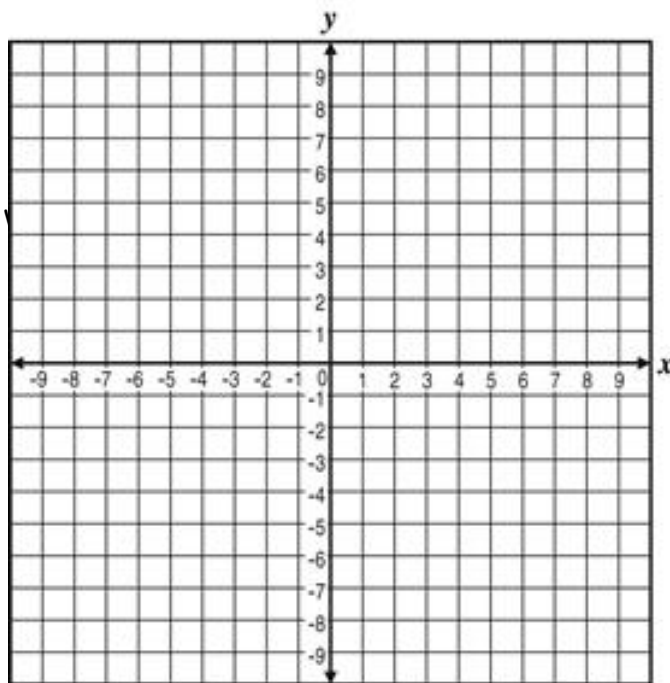
Domain

Range

Vertical Asymptote(s)

Horizontal Asymptote

End Behavior



Analyze the Behavior of the graph at  $x=-3$  and  $x=3$  also discuss and left and right hand limits

# Calculus 1.1 Understanding Limits Numerically and Graphically

Proper Limit Notation

PROPER LIMIT NOTATIONS		
TYPE OF LIMIT	PROPER NOTATION	VERBALLY:
Right-hand limit		
Left-hand limit		
General limit		

Limit Existence Theorem

When finding limits, ask yourself, "What is happening to  $y$  as  $x$  gets close to a certain number?"  
You are finding the  **$y$ -value** for which the function is approaching as  $x$  approaches  $c$ .

### LIMIT EXISTENCE THEOREM:

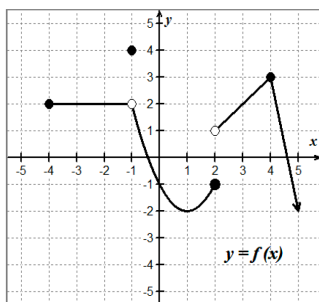
$$\lim_{x \rightarrow c} f(x) \text{ exists if and only if } \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = L$$

where  $L$  is a real number.

**Verbally:** The limit as  $x$  approaches  $c$  on  $f(x)$  will exist if and only if the limit as  $x$  approaches  $c$  from the left is equal to the limit as  $x$  approaches  $c$  from the right.

### Example 2:

Evaluating Limits from graphs



A.  $f(2)$

---

B.  $f(-1)$

---

C.  $\lim_{x \rightarrow 4^-} f(x)$

---

D.  $\lim_{x \rightarrow 2^+} f(x)$

---

E.  $\lim_{x \rightarrow 2^-} f(x)$

---

F.  $\lim_{x \rightarrow -1^+} f(x)$

---

G.  $\lim_{x \rightarrow -1^-} f(x)$

---

H.  $\lim_{x \rightarrow -4^+} f(x)$

---

I.  $\lim_{x \rightarrow -4^-} f(x)$

---

J.  $\lim_{x \rightarrow -1} f(x)$

---

K.  $\lim_{x \rightarrow 2} f(x)$

---

L.  $\lim_{x \rightarrow 5} f(x)$

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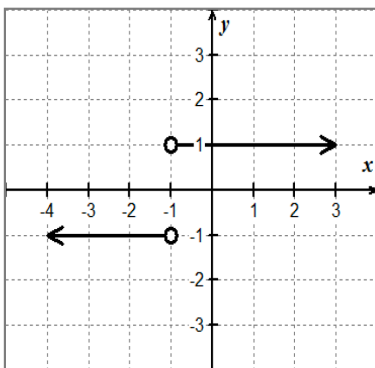
M.  $\lim_{x \rightarrow 0} f(x)$

N.  $\lim_{x \rightarrow 1} f(x)$

# Calculus 1.1 Understanding Limits Numerically and Graphically

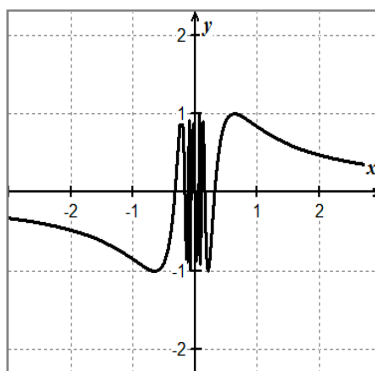
Limits can fail to exist in one of three ways

CASE 1: \_\_\_\_\_



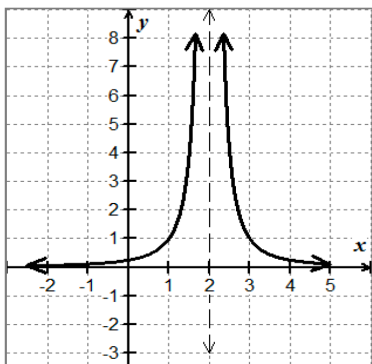
Justify why the limit does not exist at  $x = -1$  for  $f(x) = \frac{|x+1|}{x+1}$

CASE 2: \_\_\_\_\_



Justify why the limit does not exist at  $x = 0$  for  $f(x) = \sin\left(\frac{1}{x}\right)$

CASE 3: \_\_\_\_\_



Justify why the limit does not exist at  $x = 2$  for  $f(x) = \frac{1}{(x-2)^2}$

## Example 3: Evaluating Limits from Tables

$$f(x) = \frac{x-3}{x^2+2x-15}$$

$x$	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$							

Based on your analysis, what are the values of each of the limits below?

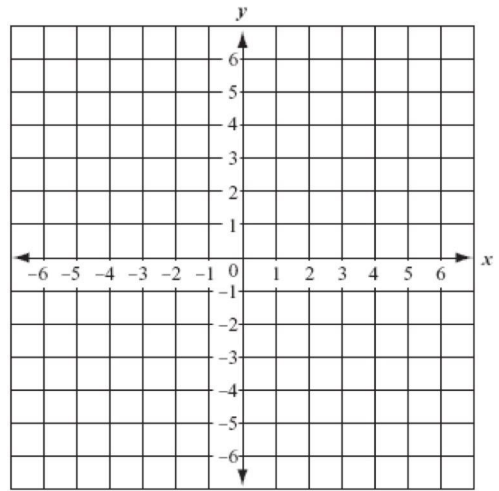
$\lim_{x \rightarrow 3^-} f(x) =$	$\lim_{x \rightarrow 3^+} f(x) =$	$\lim_{x \rightarrow 3} f(x) =$
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# Calculus 1.1 Understanding Limits Numerically and Graphically

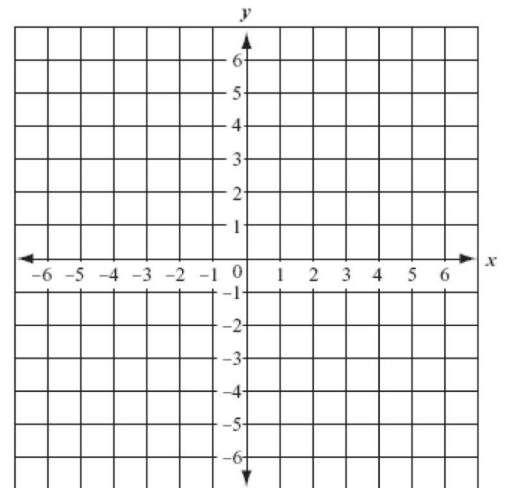
## Example 4 :

Sketch a Graph to satisfy each set of conditions

1.  $f(a)$  is undefined
2.  $x = a$  is a point discontinuity
3.  $\lim_{x \rightarrow a} f(x)$  exists



1.  $\lim_{x \rightarrow a} f(x)$  DNE
2.  $x = a$  is a jump discontinuity
3.  $f(a)$  is defined



Study Notes