Calculus 1.2 Day 2: Finding Limits Analytically

Methods to Analyze Limits	1. Direct substitution. 2. Basic Limit Theorems 3. Factor, cancellation technique. Then go back to step 1. 4. The conjugate method, rationalize the numerator. Then, go back to step 2. 5. Special trig limits of $\lim_{x\to 0}\frac{\sin x}{x} = 1$ or $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$ 6. L'Hospital's Rule (presented in Unit 3)
Substitution Theorem	If f is a polynomial function or rational function, then $\lim_{x\to c} f(x) = f(c)$ provided that if f is a rational function the value of the denominator does not equal 0. ***Always try DIRECT SUBSTITUTION first.*** *If you get $^0/_0$ or $^\infty/_\infty$ the goal will be to simplify the expression using algebraic techniques and then try substitution again.*
Example 1 Find the limits by direct substitution	**A.** $\lim_{x\to 2}(3x^2 - 5x + 4)$ **B.** $\lim_{x\to 2}\frac{x^3+1}{x+1}$ **C.** $\lim_{x\to e}\frac{\ln x}{3x}$ **D.** $\lim_{x\to 4}\sqrt[3]{x+4}$ **E.** $\lim_{\theta\to\frac{\pi}{6}}\sin 2\theta$ **F.** $\lim_{x\to 5}\log_3(x+4)$
Types of Discontinuities	What is the process for finding discontinuities of a rational function from Pre-calculus?

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You can perform the same algebraic analysis to find the limit of the removable, or point discontinuities and the non-removable, or infinite discontinuities using what we will call the **Factoring Method or Cancellation Technique**.

$$\lim_{x\to 2} \frac{x^2+x-6}{x^2-6x+8}$$

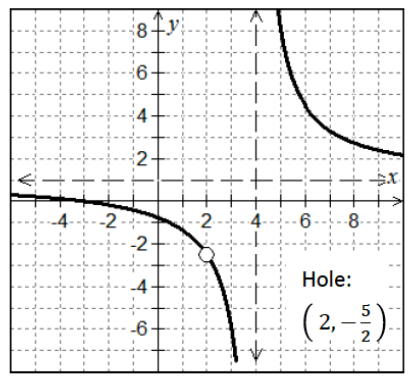
Connecting Multiple Representations of Limits

Graphically looking at the function, we see that just because it is undefined at a specific x-value doesn't mean that we can't find the limit. *Remember the Hoover Dam construction example.* Use the graph of the function to determine the value of each limit below.

$$\lim_{x\to 2} \frac{x^2+x-6}{x^2-6x+8} = \_\_\_\_\_\_\_\_\_\_$$

$$\lim_{x\to 4^+} \frac{x^2+x-6}{x^2-6x+8} = \_\_\_\_\_\_\_\_\_\_$$

$$\lim_{x\to 4^-} \frac{x^2+x-6}{x^2-6x+8} = \_\_\_\_\_\_\_\_\_\_$$



Algebraically Finding Limits of Functions at Undefined Values

Consider what happens when you try to evaluate this limit using direct substitution.

Finding one-sided limits

$$\lim_{x\to 4^+} \frac{x^2+x-6}{x^2-6x+8} =$$

As $x \to 4^+$, pick a value to the right of 4, then analyze the simplified function:

$$\lim_{x\to 4^+} f(x) =$$

$$\lim_{x\to 4^-} \frac{x^2+x-6}{x^2-6x+8} =$$

As $x \to 4^-$, pick a value to the left of 4, then analyze the simplified function:

$$\lim_{x\to 4^-} f(x) =$$

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Example 2

Finding limits of piecewise functions analytically

A. $\lim_{x\to 1} g(x)$ given, $g(x) = \begin{cases} x^3+1, & x>1 \\ x+1, & x\le 1 \end{cases}$

B. $\lim_{x\to 2} h(x)$ given, $h(x) = \begin{cases} x^2-4x+7, & x\le 2 \\ -x^2+4x-1, & x>2 \end{cases}$

Example 3

The factoring or cancellation technique

A. $\lim_{x\to 2} \frac{x^2+x-6}{x^2-6x+8}$

B. $\lim_{x\to 3} \frac{x^3-27}{x^2+x-12}$

C. $\lim_{x\to 2} \frac{x^3-2x^2+5x-10}{x^3+2x^2-13x+10}$

Example 4

The LCM/LCM Methods for Complex Fractions

A. $\lim_{x\to 0} \frac{x}{\frac{1}{4}+\frac{1}{x-4}}$

B. $\lim_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$

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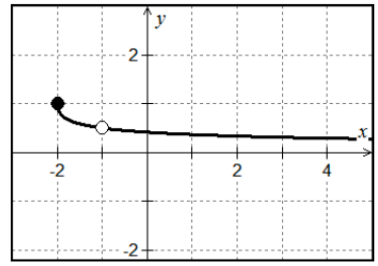
Example 5

Finding limits by rationalizing or the conjugate method

The technique of rationalization can be used to find the limit when there is a radical in the numerator or denominator.

A. The graph of $g(x) = \frac{\sqrt{x+2}-1}{x+1}$ is shown at right.

$$\lim_{x \to -1} \frac{\sqrt{x+2}-1}{x+1}$$



B. $\lim_{x \to 5} \frac{x-5}{3-\sqrt{x+4}}$

Practice

Find the limit of each function analytically

A. $\lim_{x \to -2} x^3 + 3x^2 - 4x + 5$

B. $\lim_{x \to 2^+} \frac{3x^2 - 7x + 2}{x^2 - 4}$

C. $\lim_{x \to 3} (5x+1)^{\frac{2}{3}}$

D. $\lim_{x \to -4} \frac{2x^2 + 7x - 4}{x^2 - x - 20}$

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E. $\lim\limits_{x \to 3} \dfrac{\sqrt{x+1} - 2}{x - 3}$

F. $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x}$

G. $\lim\limits_{x \to -3^-} \dfrac{x^2|2x + 6|}{4x + 12}$

H. $\lim\limits_{x \to 3} \dfrac{4x - 12}{\sqrt{x^2 - 6x + 9}}$

J. $\lim\limits_{x \to 0} \dfrac{\frac{1}{x} + \frac{1}{x+3}}{x}$

K. $\lim\limits_{h \to 0} \dfrac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$

Notes