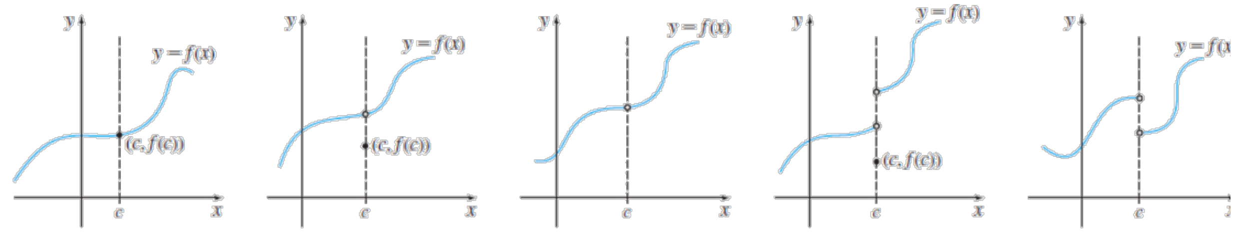
Calculus 1.3 Day 1: Continuity

Homework:

Definition of Continuity



(a) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$

(b) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) \neq f(c)$

(c) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$, $f(c)$ is not defined.

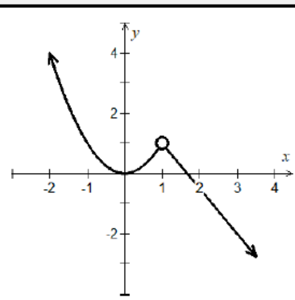
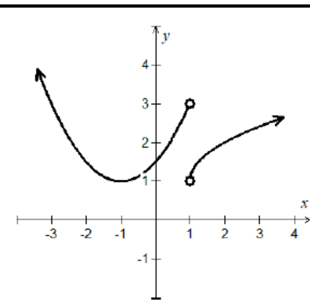
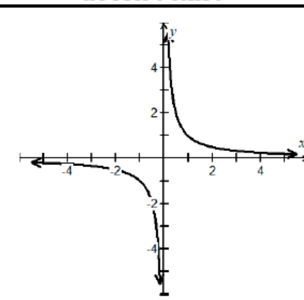
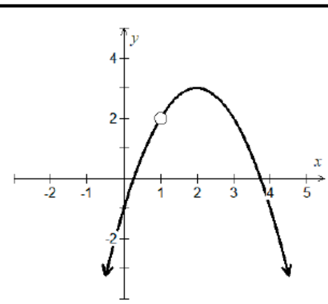
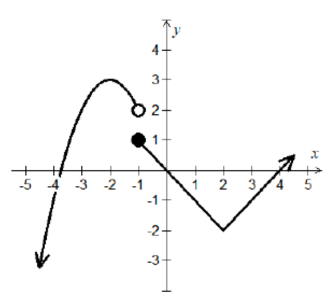
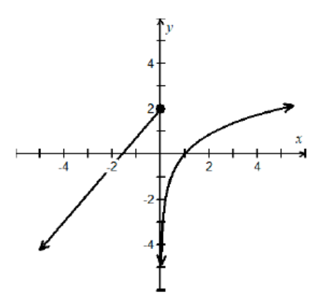
(d) $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$, $f(c)$ is defined.

(e) $\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$, $f(c)$ is not defined.

Definition of Continuity

Example 1

Classifying Discontinuities

Removable or Point (Holes) 2-sided limit exists	Non-Removable	
	Jump 1-sided limits exists	Infinite At least one of the 1-sided limits doesn't exist
		
		

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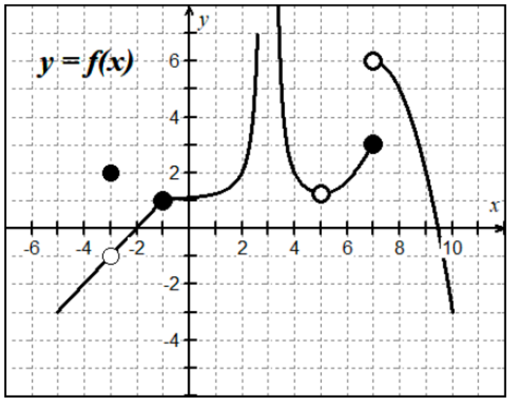
Example 2

EX #2: Find the points (intervals) at which the function is continuous, and the points at which the function is discontinuous on the interval $-5 < x < 10$.



Example 3

More Facts and Theorems

One-Sided Continuity

A function $f(x)$ is called

- ❑ Left-continuous at $x = c$ if $\lim_{x \to c^-} f(x) = f(c)$
- ❑ Right-continuous at $x = c$ if $\lim_{x \to c^+} f(x) = f(c)$

Continuity at a Point

Suppose $f(x)$ is defined on an open interval containing $x = c$.
Then f is continuous at $x = c$ if

$$\lim_{x \to c} f(x) = f(c)$$

Continuity on an Open Interval

A function is continuous on an open interval (a, b) if it is continuous at each point in the interval.

Continuity on a Closed Interval

A function is continuous on a closed interval $[a, b]$ if it is continuous on the open interval (a, b) and the function is continuous from the right at a and continuous from the left at b.

Continuity Laws of Some Basic Functions

- ❑ Polynomial functions $P(x)$ are continuous over reals
- ❑ Rational Functions $P(x)/Q(x)$ is continuous on its domain such that $Q(c) \neq 0$.
- ❑ $y = x^{1/n}$ is continuous on all reals if n is odd and continuous on $[0, \infty)$ if n is even.
- ❑ $y = \sin x$ and $y = \cos x$
- ❑ $y = b^x$ is continuous for $b > 0, b \neq 1$
- ❑ $y = \log_b x$ is continuous for $x > 0, b > 0, b \neq 1$
- ❑ Inverse functions - if $f(x)$ is continuous on an interval with range R and $f^{-1}(x)$ exists, then $f^{-1}(x)$ is continuous on domain R.

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More…

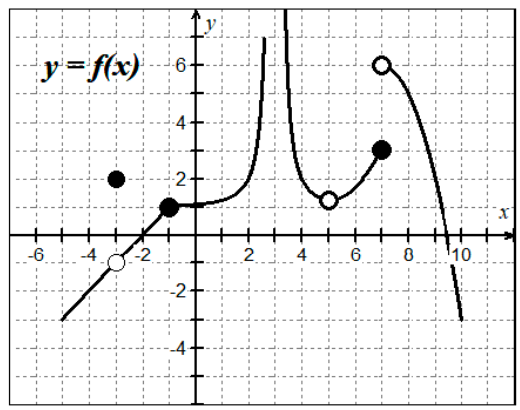
Properties of Continuity

Given functions f and g continuous at $x = c$, then the following functions are also continuous at $x = c$.

1. Scalar multiple: $b \cdot f$
2. Sum or difference: $f \pm g$
3. Product: $f \cdot g$
4. Quotient: $\frac{f}{g}$; if $g(c) \neq 0$
5. Compositions: If g is continuous at c and f is continuous at $g(c)$, then the composite function is continuous at c, $(f \circ g)(x) = f(g(x))$

Example 3

For $c = -3, c = 3,$ and $c = 7$, find $f(c)$, $\lim_{x\to c^-} f(x)$, $\lim_{x\to c^+} f(x)$, and $\lim_{x\to c} f(x)$. Justify your findings using the three-part definition of continuity.



Example 4

Find the values of x where the function is discontinuous. Describe the type, infinite or removable. Justify your answer by using the definition of continuity.

A. $f(x) = \dfrac{1}{x-3}$

B. $g(x) = \dfrac{2x^2 + 7x + 6}{x+2}$

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Example 5

Use the definition of continuity to find the value of *k* so that the function is continuous for all real numbers.

A. $g(x) = \begin{cases} kx^2, & x \le 2 \\ kx - 6, & x > 2 \end{cases}$

B. $h(x) = \begin{cases} \dfrac{|x-4|}{x-4}, & x < 4 \\ 5k - 4x, & x \ge 4 \end{cases}$

Example 6

Finding discontinuity for a piecewise function

Given $h(x) = \begin{cases} -2x - 5\ ; & x < -2 \\ 3\ ; & x = -2 \\ x^3 - 6x + 3; & x > -2 \end{cases}$ for what values of x is $h(x)$ not continuous? Justify.

Example 7

Use a system of equations to find a and b so that f(x) is continuous

Use the three-part definition of continuity to create a system of equations. Then, find the values of a and b so that $f(x)$ is continuous for all real numbers.

$f(x) = \begin{cases} 2ax - b\ ; & x < -1 \\ 6\ ; & x = -1 \\ ax^2 + b; & x > -1 \end{cases}$

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Example 8

Given

$$g(x) = \begin{cases} 2x + 1, & x < 3 \\ x^2, & x \geq 3 \end{cases}$$

find each →

A. $\lim_{x \to 3^-} g(x)$

B. $\lim_{x \to 3^+} g(x)$

C. $g(3)$

D. $\lim_{x \to 3} g(x)$

F. Is $g(x)$ continuous at $x = 3$? Justify.