Calculus 1.3: Continuity Day 3

Homework:

Intermediate Value Theorem

(IVT) is an Existence Theorem

The Big Idea:

The **IVT** says that a continuous function on an interval cannot skip values.

Using If/Then statements:

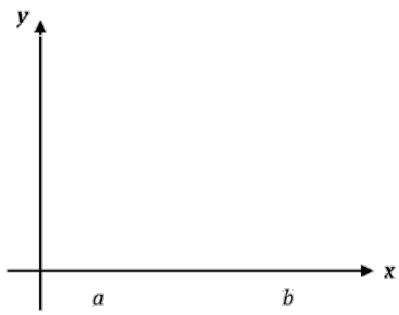
If a function $y = f(x)$ is continuous on a closed interval $[a, b]$, **Then** $f(x)$ takes on every value between $f(a)$ and $f(b)$ on that interval.

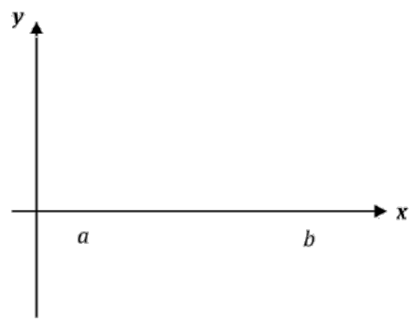
Another Approach:

Said in a different fashion, if we know a y-value, say $y = k$, that resides between the two endpoints, $f(a)$ and $f(b)$, then we are guaranteed at least one x- value, $x = c$, between the endpoints that generates that y-value, such that $f(c) = k$.

Example 1

Graphical interpretation of IVT



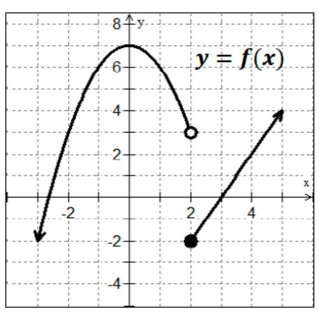


A continuous function on an interval cannot skip values.

An important outcome of I.V.T. is that it can be helpful in finding zeros of a continuous function on an interval.

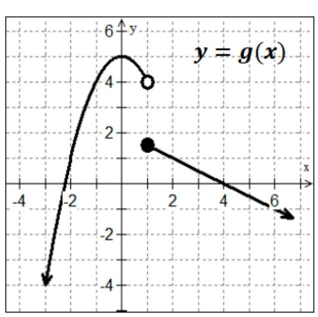
Example 2

Does IVT Apply?



A. Is there a value of $x = c$ on the interval $[-2, 5]$ such that $f(c) = 1$?

B. Does I.V.T. guarantee a value of c on the interval $[-2, 5]$ such that $f(c) = 1$? Justify.



C. Is there a value of $x = c$ on the interval $[1, 6]$ such that $g(c) = 3$?

D. Does I.V.T. guarantee a value of c on the interval $[1, 6]$ such that $g(c) = 3$? Justify.

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What three conditions are necessary to apply the Intermediate Value Theorem?

1.

2.

3.

Example 3

Apply the IVT, if possible, on $[0, 5]$ so that $f(c) = 1$ for the function $f(x) = x^2 + x - 1$.

Example 4

Use the Intermediate Value Theorem to show that $f(x) = x^3 + 2x - 1$ has a zero in the interval $[0, 1]$.

Example 5

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second is a continuous function. The table below shows selected values of the function.

t, in seconds	0	15	25	30	35	50	60
$v(t)$ in ft/sec	-20	-30	-20	-14	-10	0	10

A. For $0 < t < 60$, must there be a time t when $v(t) = -5$?

B. Justify your answer.

Calculus 1.3: Continuity Day 2

Example 6

Let $g(x)$ be a continuous function. Selected values of $y = g(x)$ are given in the table below. For which value of k will the equation $g(x) = 3/4$ have **at least two solutions** on the closed interval $[2, 8]$?

x	2	3	4	5	8
$g(x)$	3	4	k	5	2

(A) 1 **(B)** $\frac{3}{4}$ **(C)** $\frac{9}{16}$ **(D)** $\frac{3}{2}$

Example 7

For the function $f(x) = \begin{cases} (x-3)^2, x = 5 \\ 6, \quad 5 < x \leq 10 \end{cases}$. Find $f(5)$ and $f(10)$. Does IVT guarantee a y-value k on $5 \leq x \leq 10$ such that $f(5) < f(k) < f(10)$? Justify your answer.

Example 8

The functions f and g are continuous for all real numbers. The table below gives values of the functions at selected values of x The function h is given by $h(x) = g(f(x)) + 2$. Explain why there must be a value w for $1 < w < 6$ such that $h(w) = 0$.

x	1	2	6	8
$f(x)$	2	9	8	13
$g(x)$	3	-12	5	28