Calculus 1.4: Limits of Transcendental Functions - Day 1

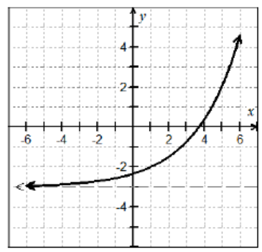
Homework

Example 1

Analyzing Limits of Exponential Functions

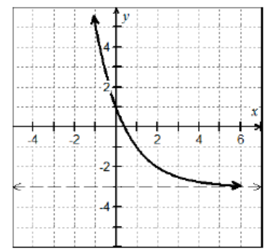
Recall that exponential equations are written in the form $y = ab^{(x-h)} + k$. You will need to find limits of exponential functions without the aid of a graph or calculator in this course. Do you remember the rules for transformations of exponential functions? Evaluate the limits using the graphs and look for patterns.

A. $f(x) = \left(\frac{3}{2}\right)^{x-1} - 3$



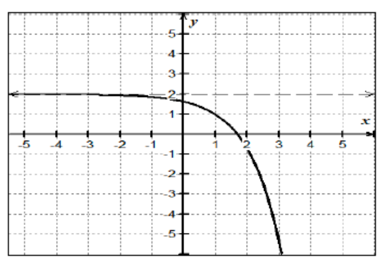
$\lim\limits_{x\to-\infty} f(x) =$ $\qquad$ $\lim\limits_{x\to\infty} f(x) =$

B. $f(x) = \left(\frac{1}{2}\right)^{x-2} - 3$



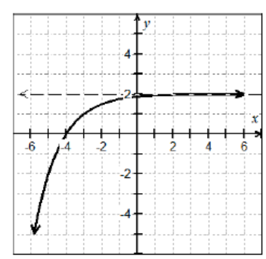
$\lim\limits_{x\to-\infty} f(x) =$ $\qquad$ $\lim\limits_{x\to\infty} f(x) =$

C. $f(x) = -e^{x-1} + 2$



$\lim\limits_{x\to-\infty} f(x) =$ $\qquad$ $\lim\limits_{x\to\infty} f(x) =$

D. $f(x) = -\left(\frac{1}{2}\right)^{x+3} + 2$



$\lim\limits_{x\to-\infty} f(x) =$ $\qquad$ $\lim\limits_{x\to\infty} f(x) =$

Complete the table and look for a pattern

Now, use the graphs from the previous example to complete the table below:

Graph of $f(x)$	Growth/Decay	Number of Reflections	$\lim\limits_{x\to-\infty} f(x)$	$\lim\limits_{x\to\infty} f(x)$
$f(x) = \left(\frac{3}{2}\right)^{x-1} - 3$				
$f(x) = \left(\frac{1}{2}\right)^{x-2} - 3$				
$f(x) = -e^{x-1} + 2$				
$f(x) = -\left(\frac{1}{2}\right)^{x+3} + 2$				

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Example 2

EX #2: WHAT JUST HAPPENED? Did you see that? Basically, the end-behavior of any exponential function tends toward three places.

CASE #1:	CASE #2:	CASE #3:

Example 3

EX #3: Let's summarize a few facts related to graph transformations in order to find limits without the aid of a graph. Using the information from above, write the four conditions that can occur.

	GROWTH		DECAY	
Value of b				
Number of reflections a or x				

Example 4

EX #4: You got this! Find the limits of each of the following exponential functions.

A. $\lim\limits_{x \to -\infty} -(0.3)^x + 2$

B. $\lim\limits_{x \to -\infty} e^{-x+1} - 3$

C. $\lim\limits_{x \to \infty} -3^{-x+1} - 3$

D. $\lim\limits_{x \to \infty} -\left(\frac{1}{3}\right)^{-x} + 1$

E. $\lim\limits_{x \to 2} \left(e^{x-3} - 1\right)$

F. $\lim\limits_{x \to -3} \left[\left(\frac{1}{2}\right)^{-x-1} + 5\right]$

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Example 5

The SQUEEZE Theorem

The Squeeze Theorem is a technique used to confirm the limit of a function by comparison with two other functions whose limits are known or easily computed. Consider some function $f(x)$ is "trapped between two functions" on an interval containing point c. Let $f, g,$ and h be functions defined on the interval except possibly at c itself. Then for $x \neq c$, in the interval $f(x) \leq g(x) \leq h(x)$ and, also that $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$. Then $\lim_{x \to c} g(x) = L$

EX #7: The Squeeze Theorem

A. If $1 \leq f(x) \leq x^2 + 2x + 2$, find $\lim_{x \to -1} f(x)$

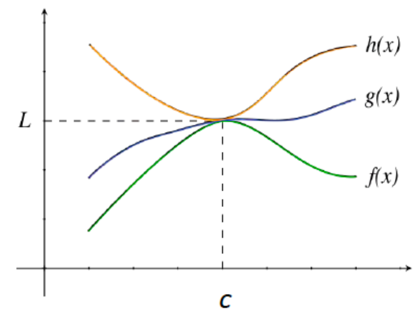
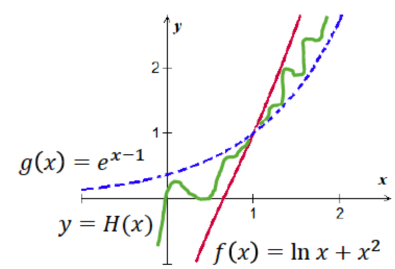


Photo: Public Domain

B. Suppose $f(x)$ and $g(x)$ are boundaries of $H(x)$, as shown on the graph below. Given $f(x) = \ln x + x^2$ and $g(x) = e^{x-1}$, for all x in the interval containing $x = 1$, except possibly at $x = 1$ itself, find $\lim_{x \to 1} H(x)$. Justify.



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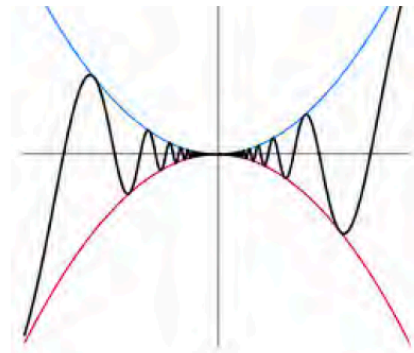
Evaluate Using the Squeeze Theorem

$$\lim_{x\to 0}\left[(x^2)\sin\left(\frac{1}{x}\right)\right]$$

A. Explain why you cannot use the Product Limit Law.



B. Consider the domain of the sine function $-1 \le \sin x \le 1$. We can conclude that $-1 \le \sin\left(\frac{1}{x}\right) \le 1$. Multiply through by x^2 to use the Squeeze Theorem.

Example 6

Find the limit. $\lim_{x\to 0}\left[(x^2)\cos\left(\frac{1}{x^2}\right)\right]$

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Example 7
Special Trig Limits

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \qquad \lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} = 0$$

A. $\lim_{\theta \to 0} \frac{\sin 2\theta}{\theta}$

B. $\lim_{\theta \to 0} \frac{\cos 3\theta - 1}{\theta}$

C. $\lim_{\theta \to 0} \frac{\sin 3\theta}{4\theta}$

D. $\lim_{\theta \to 0} \frac{1 - \cos 3\theta}{2\theta}$

Example 8
Using Algebraic Tricks

A. $\lim_{\theta \to 0} \frac{3 \sin 5\theta}{2\theta}$

B. $\lim_{\theta \to 0} \frac{1 - \cos \theta + \sin 3\theta}{\theta}$

Notes…