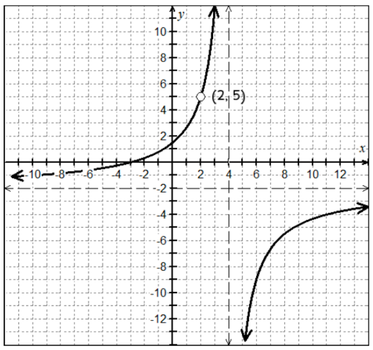
Calculus 1.5 Infinite Limits and Limits at Infinity - Day 1

Homework:

Example 1

$$f(x) = \frac{-2x^2 - 2x + 12}{x^2 - 6x + 8}$$



$\lim\limits_{x \to 4^-} f(x) =$

$\lim\limits_{x \to 1^-} g(x) =$

$\lim\limits_{x \to 4^+} f(x) =$

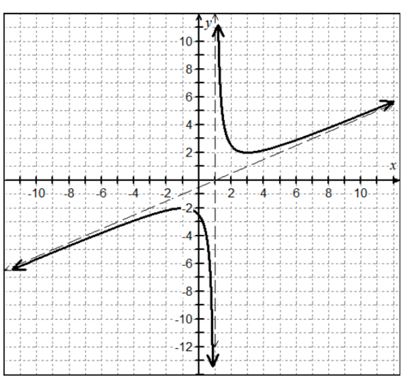
$\lim\limits_{x \to 1^+} g(x) =$

Your Discovery:

An Infinite Limit is:

Example 2

$$g(x) = \frac{x^2 - 2x + 5}{2x - 2}$$



$\lim\limits_{x \to -\infty} f(x) =$

$\lim\limits_{x \to -\infty} g(x) =$

$\lim\limits_{x \to \infty} f(x) =$

$\lim\limits_{x \to \infty} g(x) =$

Your Discovery:

A Limit at Infinity is:

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Pre-Calculus

1. When a factor cancelled from the denominator a ________________________ occurred.
2. When a factor would not cancel from the denominator a ________________________ occurred.

Example 3

Find the one-sided Limits Analytically

A. $\lim_{x \to 4^-} \frac{-2x^2 - 2x + 12}{x^2 - 6x + 8}$

B. $\lim_{x \to 4^+} \frac{-2x^2 - 2x + 12}{x^2 - 6x + 8}$

C. $\lim_{x \to 1^-} \frac{x^2 - 2x + 5}{2x - 2}$

D. $\lim_{x \to 1^+} \frac{x^2 - 2x + 5}{2x - 2}$

Definition and Justification of Vertical Asymptotes

Case #1:	**Case #2:**
$h(c) = \frac{non - zero}{zero}$	$h(c) = \frac{zero}{zero}$
$x = c$ is:	$x = c$ is:

Limit Definition (Justification) of a vertical Asymptote

In Calculus, you must use new language in order to justify!

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Example 4

Find the Vertical Asymptote(s). Justify using limits

$$h(x) = \frac{2x^2 + 9x - 5}{x^2 + 3x - 10}$$

Example 5

Limits at Infinity

using Prior knowledge, find each limit at infinity

A. $\lim_{x \to \infty} (x^2 - 4)(x^2 + 3)$

B. $\lim_{x \to -\infty} (5x^3 - 2x + 4)$

C. $\lim_{x \to \infty} \frac{3x^2 - 4}{x^2 + 1}$

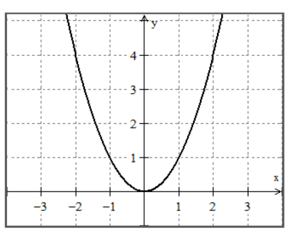
D. $\lim_{x \to -\infty} \frac{5x - 2}{x^2 + 1}$

Limits at Infinity

There are only four possible outcomes when you explore end behavior

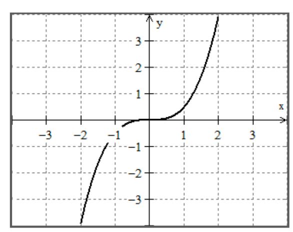
A. The curve can increase without bound.

$\lim_{x \to \infty} f(x) =$



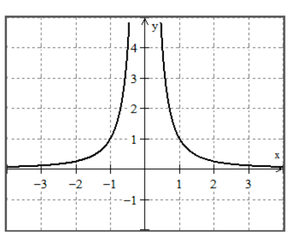
B. The curve can decrease without bound.

$\lim_{x \to -\infty} f(x) =$



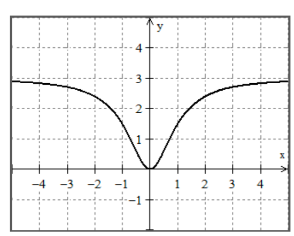
C. The curve can become asymptotic to the x-axis.

$\lim_{x \to \infty} f(x) =$



D. The curve can become asymptotic to a specific y-value.

$\lim_{x \to -\infty} f(x) =$



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Pre-Calculus to Calculus

1. If degree of numerator is less than degree of denominator (bottom heavy), then limit is zero.

$$\lim_{x \to \pm\infty} f(x) = 0$$

2. If degree of numerator equals degree of denominator (powers equal), then limit is the ratio of coefficients of the highest degree.

$$\lim_{x \to \pm\infty} f(x) = \frac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$$

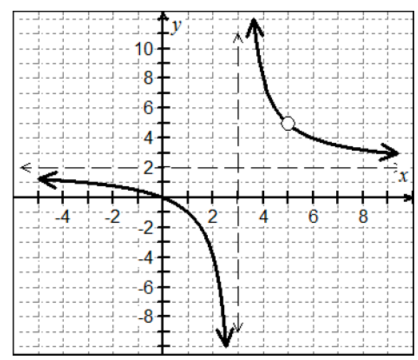
3. If degree of numerator is greater than degree of denominator (top heavy*), then limit does not exist.

$$\lim_{x \to \pm\infty} f(x) = \pm\infty$$

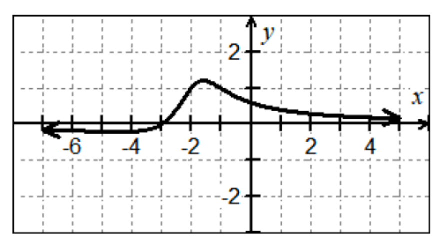
Example 6

Divide every term in the rational expression by the ***highest power of x that appears in the denominator.***
Then, apply the Properties of Limits to evaluate each "piece" to find the limit at infinity, end behavior.

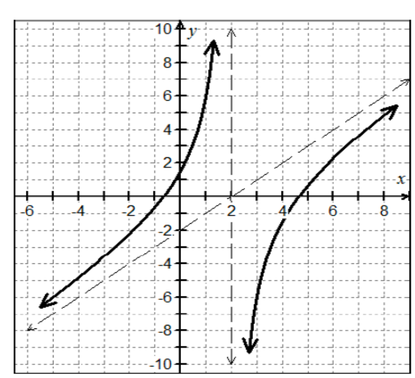
A. $\displaystyle\lim_{x \to \infty} \frac{2x^2 - 10x}{x^2 - 8x + 15}$



B. $\displaystyle\lim_{x \to \infty} \frac{x + 3}{x^2 + 4x + 5}$



C. $\displaystyle\lim_{x \to \infty} \frac{x^2 - 4x - 3}{x - 2}$



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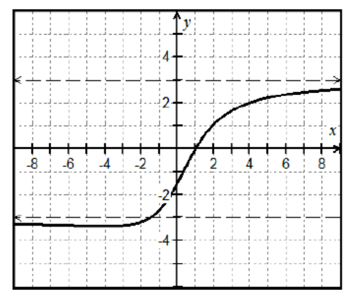
Summarize your findings from example 6	**Functions with Horizontal Asymptotes** $\lim_{x\to\infty} f(x) =$ $\lim_{x\to-\infty} f(x) =$	**Functions with Slant Asymptotes** $\lim_{x\to\infty} f(x) =$ $\lim_{x\to-\infty} f(x) =$
Limit Definition (Justification) of a Horizontal Asymptote	In Calculus, you must use new language to justify!	
Example 8 Challenge!	**CHALLENGE!** Use algebraic techniques to find the limits for $g(x) = \frac{3x-3}{\sqrt{x^2+4}}$, whose graph is shown.	 $\lim_{x\to-\infty} \frac{3x-3}{\sqrt{x^2+4}} =$ $\lim_{x\to\infty} \frac{3x-3}{\sqrt{x^2+4}} =$
NOTES		